Exercise 16

For the following exercises, the position function of a ball dropped from the top of a 200-meter tall building is given by $s(t) = 200 - 4.9t^2$, where position s is measured in meters and time t is measured in seconds. Round your answer to eight significant digits.

Compute the average velocity of the ball over the given time intervals.

a. $[4.99, 5]$

b. $[5, 5.01]$

c. $[4.999, 5]$

d. $[5, 5.001]$

Solution

The average velocity is calculated by

$$v_{\text{avg}} = \frac{s(t_2) - s(t_1)}{t_2 - t_1}.$$

Over the interval $[4.99, 5]$ the average velocity is

$$v_{\text{avg}} = \frac{s(5) - s(4.99)}{5 - 4.99} = \frac{[200 - 4.9(5)^2] - [200 - 4.9(4.99)^2]}{5 - 4.99} \approx -48.951.$$

Over the interval $[5, 5.01]$ the average velocity is

$$v_{\text{avg}} = \frac{s(5.01) - s(5)}{5.01 - 5} = \frac{[200 - 4.9(5.01)^2] - [200 - 4.9(5)^2]}{5.01 - 5} \approx -49.049.$$

Over the interval $[4.999, 5]$ the average velocity is

$$v_{\text{avg}} = \frac{s(5) - s(4.999)}{5 - 4.999} = \frac{[200 - 4.9(5)^2] - [200 - 4.9(4.999)^2]}{5 - 4.999} \approx -48.9951.$$

Over the interval $[5, 5.001]$ the average velocity is

$$v_{\text{avg}} = \frac{s(5.001) - s(5)}{5.001 - 5} = \frac{[200 - 4.9(5.001)^2] - [200 - 4.9(5)^2]}{5.001 - 5} \approx -49.0049.$$